Reply and discussion of the paper

Why the Observed Motion History of World trade Center Towers is Smooth

By Ja-Liang Le and Zdenek Bazant

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Tony Szamboti

Richard Johns

1. Introduction

In their paper, Le and Bazant respond to the claim that the motion of the roofline of the World Trade Center North Tower (WTC 1), as captured in video footage, is inconsistent with the hypothesis of gravity-driven progressive collapse. Unfortunately they do not give any sources for this claim, but it is likely that they are responding to the work of Chandler (2010) and MacQueen and Szamboti (2009).

It is agreed on all sides that the collapse of WTC 1 initiated at the 98th floor leaving a 12-story upper part to fall onto a stationary 97-story lower part, as stated by NIST NCSAR 1-6, p. 156. Le and Bazant calculate the size of the velocity reduction (during impact between the falling upper part of the tower and the stationary lower part) to be about 3%. They also find that, after impact, the upper part continues to accelerate downwards at 6.2 m/s^2. These calculations are unfortunately based on assumptions about WTC 1, especially regarding the steel columns on story 97, which are without justification, and which are contradicted by NIST.

2. Inertia Resistance

Le and Bazant first calculate the slowing of the upper portion of the building due to the inertia of the first story impacted. For reasons that unfortunately are not specified, the authors consider only the mass of the concrete floor slab to be involved in this exchange of momentum. Hence they calculate the effect of a descending mass of 54.18 Mkg striking a stationary mass of 0.627 Mkg. However, the concrete floor slab is only a small part of the floor, which includes rebar, steel decking, trusswork, and of course the live load. According to Bazant and Le (2008), from which Le and Bazant obtain the data used in their paper, m_2 = the mass of a single story is 3.87 Mkg for WTC 1. Using this value, rather than the mass of the concrete slab alone, we get a velocity ratio of 54.18/(54.18 + 3.87) = 0.93. The velocity lost is therefore about 7% of the original, rather than the 1.1% claimed. (Note that this is already more than the 3% *total* loss calculated by Le and Bazant.)

3. Column resistance

The 287 columns on the 97th story are treated by Le and Bazant as identical, even though the 47 core columns were on average much stockier than the 240 perimeter columns. The data used for a single column seem to be describing a perimeter column (stated in NIST NCSTAR 1-3D, p. 4 to be 14" square box columns) since the value M_p = 0.32 MNm may be obtained for a 14" square box column with wall thickness 6.75mm, or 0.27", according to the usual formula:

$$M_p = 1.5 \times \ b^2 t \times \ F_y$$

(b is the breadth of each flange, t is the flange thickness, and F_y is the yield stress, assumed by Le and Bazant to be 0.248 GN/m^2.)

This flange thickness 0.27" is roughly consistent with the NIST NCSTAR 1-3D report, which states that "As the elevation in the building increased, the thickness of the plates in the columns decreased, but the plates were always at least 0.25 thick". (p. 5)

The first error is then revealed when we apply this column specification, implicitly used by Le and Bazant, to calculate the total cross-sectional area of the columns. We then obtain a total area A = 2.75m^2, for the 287 columns, which is much less than the authors' own value of 6.05m^2. One is bound to wonder how this value of 6.05m^2 was obtained, since no reference or calculation is given for it. We shall show below that the correct value is roughly A = 2.3 (perimeter) + 1.7 (core) = 4m^2.

The authors' second error is to use a value of F_y = 0.248 GN/m^2 (36 ksi) for the yield stress of the columns on the 97th story. This is incorrect, as thin-walled perimeter columns on the upper stories are reported by NIST to be 55ksi – 100ksi (NCSTAR 1-6, p. 61, and NCSTAR 1-3B, Table 4-2, p. 52). We will conservatively estimate the average yield stress to be 65ksi, i.e. 0.45 GN/m^2. Since the formula for M_p is linear with the yield stress F_y, correction of this error increases the value of M_p for the perimeter columns to 0.58 MNm. This is a very conservative estimate, since NIST reports the actual yield stresses to be above the nominal ones. (NCSTAR 1-6, p. 61) We see that the authors' estimate of 0.32 MNm is hardly an upper bound.

The calculation of M_p for the core columns is laborious, since the columns are a variety of sizes and steel types. They are wide-flange columns, with flange dimensions ranging from 16.695" x 3.033" down to 8" x 0.528", and either 36, 42, 45, or 50 ksi. (See the publicly available NIST SAP2000 model data, reproduced by MacQueen and Szamboti (2009), pp. 22-3.) The M_p values range from 2.01 MNm down to 0.09 MNm, with the average being 0.75 MNm. Again, this is far above the authors' estimate of 0.32 MNm.

With these corrections in place, let us calculate the total yield load for all the columns. First the 240 perimeter columns: P = 240 x 0.00675 x 4 x 0.3556 x 0.45 x 10^9 = 1.04 GN.

The calculation for the core is more laborious, due to the variation in column dimensions and yield stress. But using the same columns data, the total cross-sectional area of the core columns is found to be 1.69 m^2, and the maximum load is 0.46 GN.

Using these corrected values, we can calculate the load-displacement curve. For this we also need the column length, L, which is 3.7m in the case of the core columns, and 2.3m for the perimeter columns, due to the 1.4 m deep spandrel plates. The resistive force F_b is given by the formula below, where the number of columns is *n*, and *u* the reduction in column length.

$$F_b = \frac{4nM_p}{L\sqrt{1-\left[1-\left(\frac{u}{L}\right)\right]^2}}$$

Adding the two resistive forces, due to the perimeter and core columns, we get the graph shown in Fig. 1.

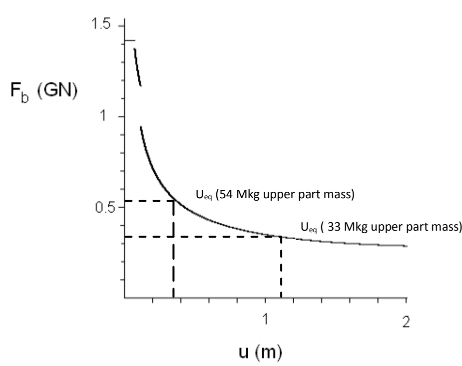

Fig. 1. Diagram of load vs. displacement during axial deformation and buckling

By inspecting this graph we see that u_{eq}, the displacement at which the column resistance equals the 0.53 GN weight of the upper part (i.e. the 54 Mkg mass used by Le and Bazant) is roughly 0.38m, rather than the 0.065m claimed.

Up to this point we have used Le and Bazant's mass value of 54 Mkg for the upper part of the tower, but this is probably an overestimate since it conflicts with the data provided in the NIST WTC report concerning their description of the floor structures, total steel weight found in contracts, and live and superimposed dead loads. A more reasonable estimate, based on these data, is 33 Mkg for the 12-story upper part, i.e. 2.75 Mkg per story. This lower estimate is also much closer to typical mass per square meter values for other buildings sharing this type of construction, such as the Sears Tower and John Hancock building. For a detailed treatment of these arguments, see Urich (2007).

From here on, therefore, we shall calculate using the 33 Mkg value as well as Le and Bazant's 54 Mkg. For example, using the lower mass value, u_{eq} occurs at roughly 1.12m as shown in Fig. 1.

4. Calculating the Velocity Curve

In order to verify the accuracy of the gravity-driven model, we shall calculate the velocity curve for the roof line, and compare it with the behavior of WTC 1 itself. Fortunately there is high-resolution footage of the collapse of WTC 1 shot by professional filmmaker Etienne Sauret, and used for the documentary film *WTC - The First 24 Hours* (2002). Each pixel of this footage represents 0.27m of the tower, and the frame rate is 30 per second, allowing for very accurate measurements of the motion.

David Chandler, one of the "internet" sources that Le and Bazant presumably refer to, has analyzed this motion using Tracker, an open source video analysis tool. His graph is shown below, together with two velocity plots for a gravity-driven collapse.

The calculated velocity of the roofline was obtained numerically using the load-displacement curve shown above. We also assumed Le and Bazant's freefall acceleration during the collapse of the first story, and the two possible mass values, as mentioned above. The floors are treated as rigid and incompressible, so that no energy is lost deforming them, even though in reality this would be a significant energy drain. The upper part of the building is also modeled as a rigid block, which Le and Bazant regard as a reasonable approximation.

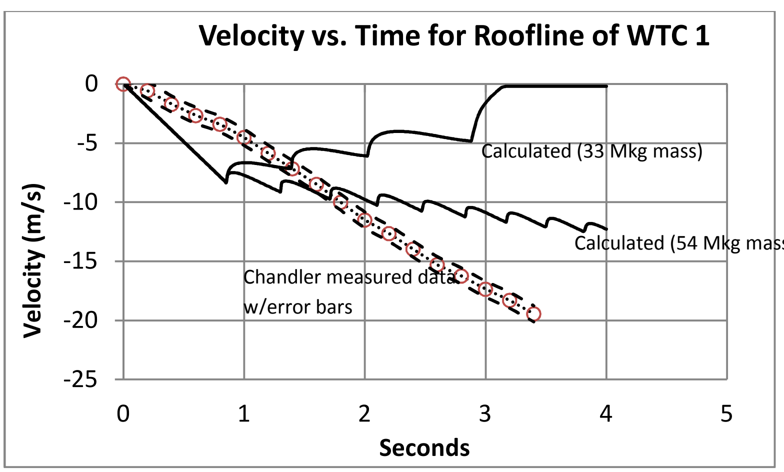

Fig. 2. Measured and calculated velocity curves

It is questionable whether the velocity fluctuations seen on the graph in Fig. 2 (using the 54 Mkg mass value claimed by Le and Bazant) would be visible on the video, since the measurement error is $\pm$0.675 m/s. But it is clear that the calculated average downward acceleration is much less than the observed value.

With the 33 Mkg mass the calculated velocity decrease is roughly 2 m/s, and should be visible in a velocity plot obtained from the Sauret video footage. Also, the average acceleration after impact is negative (i.e. upward), which would be easy to observe.

5. Conclusion

The analysis of Le and Bazant, while sound theoretically, uses incorrect input values. These errors each have the effect of reducing the resistance of the lower part of the building. As a result, their calculated velocity drop on impact is too low, and the calculated acceleration following that drop is too high.

References

Chandler, D. (2010). "Destruction of the World Trade Center North Tower and Fundamental Physics", *Journal of 9/11 Studies*, available at http://www.journalof911studies.com.

MacQueen, G., and Szamboti, T. (2009). "The Missing Jolt: A Simple Refutation of the NIST-Bazant Collapse Hypothesis", *Journal of 9/11 Studies*, available at http://www.journalof911studies.com.

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Urich, G. (2007). "Analysis of the Mass and Potential Energy of World Trade Center Tower 1", *Journal of 9/11 Studies*, available at http://www.journalof911studies.com.

Rebuttal to Criticisms of Reviewer #2

Richard Johns
Anthony Szamboti

June 7, 2012

The full text of the reviewer's comments, as provided to us over email, including quotations from our discussion, are shown below in 10-point Arial font, indented. Our responses are in Times font.

> Reviewers' comments:
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> AE: On the basis of the enclosed review, the paper is declined for the lack of substantive arguments in terms the underpinning (e.g. tower velocity) calculations.
>
> Reviewer #2: The Jan 2011 technical note (TN) by Le and Bazant discussed how the upper portion of the WTC towers fell and impacted the remaining building section below, with a focus on the mechanics used to determine the velocity of the upper portion as it impacted the section below and the effect of degradation on the velocity. The change in velocity at impact was shown to be too small to detect on available videos. This paper builds on a series of papers in the Journal of Engineering Mechanics, and the entire sequence of papers needs to be considered by the discussion authors.
>
> The discussion paper by Szambati and Johns asserts that the input values used for the calculations of velocity were incorrect. Therefore, the levels of computed deceleration at impact and acceleration following impact are thought to be incorrect.
>
> However, as noted below, the authors have not successfully demonstrated their concerns because they have not accurately represented the work by Le and Bazant or presented the basis for the input values they feel are correct.
>
> The reviewer has the following comments about the discussion paper:
>
> 2. Inertia Resistance
>
> The authors stated that the reasons for only using the concrete mass are not stated. However, Le and Bazant reference their 2008 paper for the source of values used, and the authors go on to use values from that paper. Le and Bazant (2008) define mc as the "mass of one floor slab". A floor slab is terminology often used to refer to the constructed floor, not just the concrete.

Response: No doubt the term 'floor slab' is sometimes used this way, but not in this case. The mass used by Le and Bazant, 0.627 Mkg, cannot be the mass of the entire constructed floor, since the latter (including the live load) is at least 2 Mkg. A very rough calculation of the mass of a lightweight concrete slab, 11cm thick, and roughly 60 by 60 metres, density 1750 kg/m^3, is about 0.7 Mkg. Of course there was no floor in much of the building core, which no doubt accounts for the small difference between this value and Le and Bazant's.

The authors use the m2 value defined by Le and Bazant as "mass of a single story", which includes the steel columns and floor slab, in a mass ratio of the upper section mass (M) to (M+m2). M/(M+m2) cannot be equated to the velocity reduction in equation 2 in the TN.

The authors statement below is incorrect:

"The velocity lost is therefore about 7% of the original, rather than the 1.1% claimed. (Note that this is already more than the 3% total loss calculated by Le and Bazant.)"

The 1.1% velocity reduction by Le and Bazant was based on rigid mass interactions in equation 2, and the 3% velocity reduction was based on deformation and interaction of both masses in equation 11.

This criticism is baffling to us. Our velocity reduction calculation, based on the inertia of floor 97, does not depend on the floor being rigid. It is simple Newtonian physics. When a body of mass $14m$ strikes a stationary one of mass m, and they stick together, the resulting body has mass $15m$ and has 14/15 = 0.93 of the original velocity. This follows from the conservation of linear momentum, which applies to all collisions, regardless of the rigidity of the bodies involved. If the bodies are compressible, then the velocity reduction is spread over a longer time interval, but the size of the reduction is unaffected. We can see no reason at all to suppose that only the concrete slab would be accelerated by the impact, rather than the whole floor assembly. Neither Le and Bazant nor Referee #2 has supplied such a reason.

3. Column Resistance

The authors state:

"The 287 columns on the 97th story are treated by Le and Bazant as identical, even though the 47 core columns were on average much stockier than the 240 perimeter columns. The data used for a single column seem to be describing a perimeter column (stated in NIST NCSTAR 1-3D, p. 4 to be 14" square box columns) since the value Mp = 0.32 MNm may be obtained for a 14" square box column with wall thickness 6.75mm, or 0.27", according to the usual formula:

Mp = 1.5 x b2t x Fy

(b is the breadth of each flange, t is the flange thickness, and Fy is the yield stress, assumed by Le and Bazant to be 0.248 GN/m2)."

The column data used by Le and Bazant was representative section for all of the core and

> perimeter columns, as described in Le and Bazant (2008) under Variation of Mass and Buckling Resistance Along Height section.

The section referred to does contain some information about the columns, but it does not describe any single column spec that is representative for the columns between floors 97 and 98. Interestingly, it does give 10mm as the web thickness for the perimeter columns on the aircraft impact level. Using 10mm with the other parameters (breadth 0.3556m and yield stress 250 MPa) gives M_p = 0.448 MNm rather than 0.32 MNm, so it could not have been used in Le and Bazant (2011). In our opinion, Le and Bazant's TN should have stated clearly, in the paper itself, their assumed specs for the columns on story 97. As it is, we are forced to guess these specs, based on the few numbers they do supply, such as the plastic moment.

> The plastic moment, Mp=0.32 MNm is the "average yield bending moment of one column" for "n=287 columns (approximately considered as identical)". Identical does not imply that they are all perimeter box columns.
>
> Further, it is not clear what 1.5 x b2t x Fy represents in the Mp equation, as it is not an expression for the plastic modulus of either a hollow box section or a wide-flange section about the plastic neutral axis. The authors need to give a source for the equation.

Our equation for M_p is a simplified version of the one given in:

Gaylord E. H. and Gaylord C. N. (1979) *Structural Engineering Handbook*, McGraw-Hill.

On page 7-3 the plastic section modulus is given for a hollow rectangular section with external dimensions *b* x *d*, and flange/web thicknesses *t* and *w* as:

$$Z_p = \frac{bd^2}{4}\left(1 - \left(1 - \frac{2w}{b}\right)\left(1 - \frac{2t}{d}\right)^2\right)$$

For a hollow square section, with equal flange and web thicknesses, we put $d = b$ and $w = t$ to get:

$$Z_p = \frac{b^3}{4}\left(1 - \left(1 - \frac{2t}{b}\right)^3\right)$$

We then derived a simplified formula for thin-walled sections where t << b. Multiplying out the brackets and dropping terms containing t^2 and higher orders, one obtains:

$$Z_p \approx \frac{3}{2} b^2 t$$

When this is multiplied by F_y it gives the formula for M_p stated in our discussion. No doubt the use of the simplified formula was a stumbling block to the reviewer, and it also gives slightly different M_p values from the exact one. We would be happy to use the exact formula instead.

> Given the comments above, the 'first error' cited by the authors as an incorrect total cross-sectional column area for a floor is not persuasive. Le and Bazant used a representative section (noted above) and there is no basis for the author's assertion that A= 4 m2.

The value A = 4m^2 is obtained by adding 2.3m^2 (perimeter) to 1.7m^2 for the core. The total cross sectional area for the (roughly square) perimeter columns was calculated as 240 (columns) x 4 x 0.3556m (breadth) x 0.00675m (thickness). The total cross sectional area for the core columns was obtained by adding the cross sectional area for each core column, as given in the NIST SAP2000 model data.

> The noted 'second error' of the Fy value could not be verified.
>
> "The authors' second error is to use a value of Fy = 0.248 GN/m2 (36 ksi) for the yield stress of the columns".
>
> I did not find it in the 2011 technical note, or in the other papers by Le and Bazant. Le and Bazant did account for varying Fy of the columns in their representative section.

Le and Bazant did indeed use F_y = 250 MPa, i.e. 0.25 GN/m^2. While it is not explicitly stated in their 2011 paper, it can be calculated from their Equation (3). They call it σ_0, and it equals $(1.513 \times 10^9)/6.05 = 0.25 \times 10^9$. Bazant and Le also give this value explicitly in their 2008 closure to G. Szuladzinski's discussion (JEM 2008, p. 921).

> For the calculation of Mp, I looked at the referenced MacQueen and Szamboti (2009), which listed column Fy and dimensions for core columns, but did not list any plastic moment values. Given the Mp equation above, the values listed for are suspect.

It is disappointing that the reviewer finds our M_p values to be "suspect" without actually checking any of them. All the necessary data to do so are provided in the supplied MacQueen and Szamboti reference. Each flange has plastic section modulus $t.b^2/4$, so the total is $t.b^2/2$ for the two flanges. (Here we neglected the small contribution from the web, i.e. $\frac{1}{4}(d - 2t)w^2$, where $d - 2t$ is the web length and w the thickness. The full formula is given in Gaylord and Gaylord text referenced above, p. 7-3.)

In our discussion we stated the M_p values calculated using this formula for the largest and smallest core columns. For example, the largest type of core column on this story has b =

16.695" = 0.424m, and t = 3.033" = 0.077m, and has a 42 ksi (290 MPa) yield stress. We then have

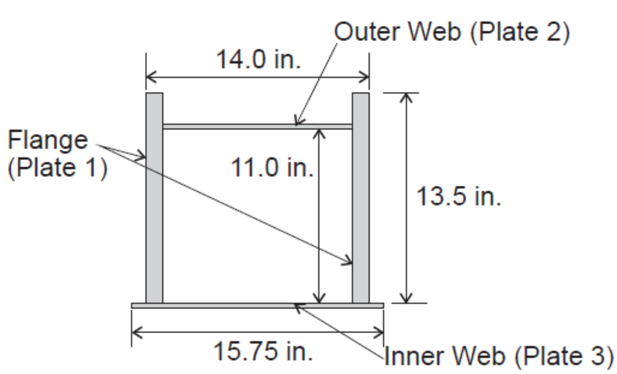
$$M_p = (0.077 \times 0.424^2 \times 290 \times 10^6)/2 = 2.01 \text{ MNm},$$

exactly as stated in our discussion. We calculated the M_p values in the same way for all of the 47 core columns using a spreadsheet, and found the average to be 0.75 MNm. If anyone doubts this figure they are welcome to calculate it for themselves. We can also provide our Excel file, upon request.

> The authors computed a total yield load for
>
> "First the 240 perimeter columns: P = 240 x 0.00675 x 4 x 0.3556 x 0.45 x 10^9 = 1.04 GN."
>
> Equations need to be presented with defined variables, and then followed by values is desired. It is not clear what 0.3556 represents, and the area of the perimeter columns included flange sections that extended beyond the 'box' section, which is not discussed or included in the calculations. Based on these points, the values listed for the core columns are also suspect, as insufficient basis for the values presented are provided.

We think this calculation is clear enough, but it would be easy to add the explanation that 0.3556m is the breadth of a perimeter column, and 0.00675m the flange thickness, so that 0.00675 x 4 x 0.3556 is the cross-sectional area of one column. Multiplying by the yield stress 0.45×10^9 N and the number of perimeter columns (240) gives the total yield load for the perimeter columns on the 97^{th} story.

The appeal to extended flange sections, to account for Le and Bazant's very high area value, is grasping at straws. The figure below is part of Fig. 2-3 on p. 7 of NIST NCSTAR 1-3A, and shows that the total XS length of the flanges and webs is 13.5" x 2 + 14" + 15.75" = 56.75". Hence our value of 14 x 4 = 56" is admittedly too low, but only by about 1.3%, which is not significant.



> The authors use the unsubstantiated values from above in an equation from Le and Bazant (2002) that computes plastic axial load Fb or a given axial shortening u.
>
> The input values for the equation include a core column length of 3.7 m and a perimeter column length of 2.3 m. Clearly, column lengths must all be the same on a given story - the spandrel plates were attached to the columns but did not act as columns. Thus, Figure 1 is incorrect.

The length of concern is the unsupported column length and it is different between the columns in the core and those on the perimeter due to the depth of the beams involved. In taking 2.3m as the unsupported length of a perimeter column we are following Bazant and Zhou (2002), p. 5, except that we measured the spandrel height to be 1.4m rather than 1.2m. This can be changed without drastically affecting the results.

> The authors go on to estimate their own value of the mass of the upper descending portion of the tower, simply based on floor densities from other high-rise buildings. While that information is interesting, it is not sufficient to claim that the correct value is 2.75 Mkg per story.

In our discussion paper we actually refer to a detailed analysis by G. Urich, which is based on the NIST reports' description of the floor structures, total steel weight found in contracts, and live and superimposed dead loads. We do not argue solely by comparison with the Sears Tower and John Hancock building, although that provides additional evidence. Moreover, we recently found that NIST NCSTAR 1-6D, p. 176, Table 4-7, directly states the actual total load on the columns between floors 98 and 99 to be 73,143 kips, i.e. roughly 33 Mkg. With the collapse initiating on the 98th floor, as referenced in NIST NCSTAR 1-6, p. 156, the falling upper section mass would be roughly 33 Mkg, as stated in our discussion. There are many separate lines of evidence leading to mass estimates in this range, while Le and Bazant provide no justification at all for their much-higher estimate. Hence our criticism is well supported and very reasonable.

> 4. Calculating the velocity curve.
>
> Given the concerns about the values for mass and column properties, the velocity computations in this section are suspect. The basis of the computed velocity curves for the 33 and 54 Mkg masses are not described. Note that in Figure 2 that the 33 Mkg mass has a zero velocity at approx. 3.2 s, well before the collapse is completed.

All the necessary input values are given, so that anyone can calculate their own curves to verify ours. We would be happy to provide hand calculations that give approximately the same results as the curves shown, which were produced numerically. We were not able to include such calculations in the original discussion, since we had reached the upper word limit.

In summary, *Reviewer #2 has not found any error at all* in our criticisms of Le and Bazant's TN. We have correctly cited the TN itself, as well as Bazant's earlier papers on the subject, and the NIST reports. Our criticisms, summarised below, are therefore still valid.

1. Le and Bazant do not adequately state their assumed specifications for the columns on story 97.

2. The values they do state, i.e. average M_p = 0.32 MNm and total XS area 6.05 m^2, are unsupported by any references or calculations, and *not even consistent with one another*, given the known number and external dimensions of the columns, their own value for the yield stress, and the standard textbook formula for M_p.

3. In calculating the momentum exchange between the falling upper block and the first stationary floor, Le and Bazant have incorrectly used the mass of the concrete slab only, rather than the full floor assembly.

4. Le and Bazant's mass value of 54.18 Mkg for floors 99-110 (plus the roof) is unsupported by any evidence, and is much greater than the 33 Mkg value given by NIST.

5. Le and Bazant's average value for the yield stress of the columns on story 97 contradicts the yield stresses provided by NIST.

6. With all these corrected data the value of u_{eq}, i.e. the downward displacement at which the resistive and gravitational forces balance, is roughly 1.12m, not the 0.065m they claim.

7. Using the corrected data, Le and Bazant's own methods predict a velocity reduction that would be visible in a velocity plot derived from Etienne Sauret's high-definition video footage of WTC 1. (Our discussion paper, unlike the TN, includes this necessary empirical data, and no such reduction is visible.) The conclusion of Le and Bazant's TN is not supported by the available evidence.

Discussion of the paper

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By Ja-Liang Le and Zdeněk Bažant

DOI: 10.1061/_ASCE_EM.1943-7889.0000198

Journal of Engineering Mechanics, Vol. 137, No. 1, January 1, 2011, pg. 82-84

Tony Szamboti

Richard Johns

1. Introduction

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2. Inertia Resistance

Le and Bažant first calculate the slowing of the upper portion due to the inertia of the first story impacted. For reasons that are not specified, they consider only the mass of the concrete floor slab to be involved in this exchange of momentum. They calculate the effect of a descending mass of 54.18 Mkg striking a stationary mass of 0.627 Mkg. However, the concrete floor slab is only part of the overall floor mass, which also includes rebar, steel decking, trusswork, and the live load. According to Bažant and Le (2008, p. 905), from which Le and Bažant obtain the data used, m_2 = the mass of a single story is 3.87 Mkg for WTC 1. Using this value, we get a velocity ratio of 54.18/(54.18 + 3.87) = 0.93. The velocity lost is therefore about 7% of the original, rather than the 1.1% claimed. (Note that this is already more than the 3% *total* loss, calculated by Le and Bažant.)

3. Column resistance

For simplicity, Le and Bažant's calculations assume that the 287 columns on the 97th story are identical. Unfortunately the full specifications of this representative column are not stated. We are told that the plastic moment M_p for this column is 0.32 MNm, and from Equation (3) we can infer that the yield stress σ_0 = 250 MPa. The total cross-sectional area of the 287 columns is stated to be 6.05 m^2. The shape of the column, its overall dimensions, and flange and web thicknesses are not given. We can find no specification consistent with this data.

Most of the columns (240 of the 287) were perimeter columns, the overall dimensions and shape of which are stated by NIST (NCSTAR 1-3D, p. 4) to be approximately 14" square box columns, i.e. having width and breadth equal to 0.3556 m. To calculate M_p we used a standard formula for the plastic section modulus of a hollow rectangular section (see Gaylord et al, 1979, 7-3), putting width equal to breadth *b*, web thickness equal to flange thickness *t*, and multiplying by the yield stress, gives:

$$M_p \ = \ \frac{b^3}{4}\left(1-\left(1-\frac{2t}{b}\right)^3\right)\sigma_0. \qquad (1)$$

Calculating backwards (from M_p=0.32 MNm) gives t = 7.02 mm. This is much less than the 10 mm thickness given in Bažant and Le (2008, p. 896) for the aircraft impact level, and even a little less than the 7.5 mm they state for the top story. It also entails a total cross-sectional area of 287 x 4 x 0.3556 x 0.00702 = 2.87 m^2, which is less than half of the 6.05 m^2 stated. The authors need to explain how their M_p value was obtained.

Our estimate of the average plastic moment of the columns on story 97 is 0.64 MNm, obtained as follows. For the perimeter columns, we conservatively assume web and flange thicknesses t = 7.5 mm. The yield stress of the perimeter columns at story 97 is reported by NIST to be 55ksi – 100ksi (NCSTAR 1-6, p. 61, and NCSTAR 1-3B, Table 4-2, p. 52). We estimate the average yield stress to be 65ksi, i.e. 450 MPa, which is also conservative, since NIST reports the measured yield stresses to be above nominal. (NCSTAR 1-6, p. 61). This gives M_p = 0.61 MNm for the perimeter columns.

The core columns vary in size and steel types. They are wide-flange columns, with flanges ranging from 16.695" x 3.033" down to 8" x 0.528", and either 36, 42, 45, or 50 ksi yield strength. (See the available NIST SAP2000 model data, reproduced by MacQueen and Szamboti (2009), pp. 22-3.) To calculate M_p for the weak axis the plastic section modulus $Z_p = ½\, t.b^2$, also obtained from Gaylord et al (1972, 7-3), was used, omitting the small contribution from the web. The M_p values for core columns were found to range from 2.01 MNm to 0.09 MNm, the average being 0.75 MNm. The weighted average, for core and perimeter columns, is then 0.64 MNm. We conclude that 0.32 MNm is much too low.

Using this corrected M_p value, together with the other column data stated above, we can repeat Le and Bažant's calculations for the velocity reduction of the upper part of WTC 1. First we calculate the total yield load for all columns. For the 240 perimeter columns: $P = 240 \times 4bt\sigma_0$ = 1150 MN. For the core, using the NIST data, the total cross-sectional area of the core columns is found to be 1.69 m^2, and maximum load is 460 MN. In total, we have P = 1,610 MN.

For calculating the load-displacement curve we also need the column length L, given by Le and Bažant as 3.7 m for all the columns. Bažant and Zhou (2002, p. 5) state the effective height of the perimeter columns to be 2.5 m, the distance between the 1.32 m deep spandrel plates, that were heavier gauge than the adjacent column webs. (See NIST NCSTAR 1-3A, pp. 7-9.) Since our aim is to calculate a conservative estimate of the velocity drop, however, we will ignore the spandrel plates and use L = 3.7 m – which makes the perimeter columns more slender, substantially reducing their resistance during buckling. The resistive force F_b is then given by the formula below (see Bažant and Zhou 2002, p. 6) where number of columns is n, and u the reduction in column length.

$$F_b = \frac{4nM_p}{L\sqrt{1-\left[1-\left(\frac{u}{L}\right)\right]^2}} \qquad (2)$$

Using Mp = 0.64 MNm we get the graph shown in Fig. 1.

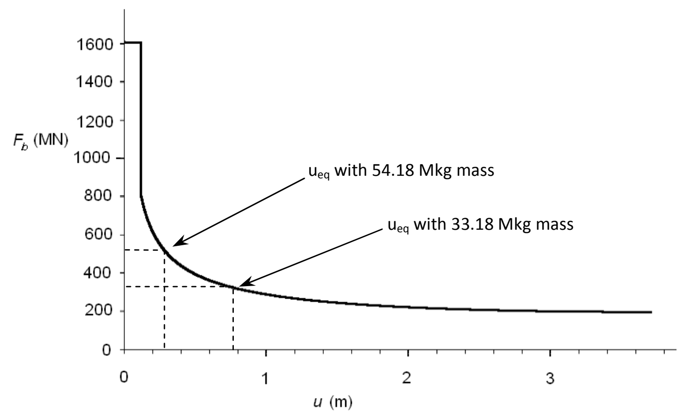

Fig. 1. Diagram of load vs. displacement during axial deformation and buckling

The average resistance of the columns is 310 MN, using numerical integration. The displacement u_{eq}, at which column resistance equals the 530 MN weight of the upper part (i.e. the 54.18 Mkg mass used by Le and Bažant) is 0.27 m, rather than the 0.065 m claimed.

Up to this point we have used Le and Bažant's mass value of 54.18 Mkg for the upper part of the tower, but this conflicts with the NIST report (NCSTAR 1-6D, p. 176, Table 4-7), which states the actual total load on the columns between floors 98 and 99 to be 73,143 kips, i.e. 325.4 MN or 33.18 Mkg. NIST's estimate is also much closer to typical mass per square meter values for other buildings sharing this type of construction, such as the Sears (now Willis) Tower and John Hancock building. For a detailed examination of the masses of WTC 1 and 2 see Urich (2007).

From here on, we will use NIST's 33 Mkg figure in our calculations. For example, u_{eq} then occurs at roughly 0.76 m, as shown in Fig. 1.

4. Calculating the Velocity Curve

To verify the accuracy of the gravity-driven model, we can calculate the velocity curve for the roof line, and compare it with the behavior of WTC 1 itself. Fortunately, there is high-resolution footage of the collapse of WTC 1 shot by professional filmmaker Etienne Sauret, and used for the documentary film *WTC - The First 24 Hours* (2002). Each pixel of this footage represents 0.27 m of the tower, and frame rate is 30 per second, allowing for accurate measurements of the motion.

David Chandler has analyzed this motion using Tracker, an open source video analysis tool. His graph is shown below, together with a calculated velocity plot for a gravity-driven collapse.

The calculated velocity of the roofline was obtained numerically using the load-displacement curve shown above, and scaling up linearly for lower stories, according to the increasing design load. We also assumed Le and Bažant's freefall acceleration during the collapse of the first story. Floors are treated as rigid and incompressible, and assumed to stick together upon impact. The upper part of the building is modeled as a rigid block, which Le and Bažant regard as a reasonable approximation.

It is easy to derive an approximation of this curve, using hand calculations, given the average 97th story column resistance of 310 MN, which is approximately NIST's (325.4 MN) weight for the upper part of the building. Hence the average velocity is approximately constant after the first impact – decreasing slightly due to the inertia of the impacted stationary floors.

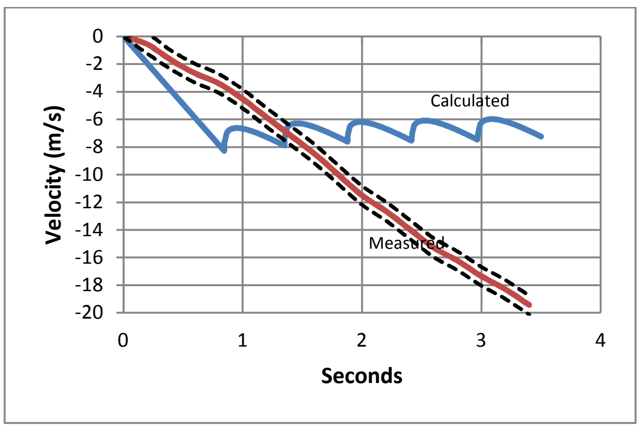

Fig. 2. Measured and calculated velocity curves

The calculated first velocity decrease is 1.65 m/s (approximately 20%), and would be visible (if it existed) in a velocity plot obtained from the Sauret video footage. Also, the predicted average acceleration after impact (roughly zero) is significantly different from what was observed.

5. Conclusion

The analysis of Le and Bažant uses incorrect input values. These errors each have the effect of reducing the resistance of the lower part of the building. As a result, their calculated velocity drop on impact is too low, and their calculated acceleration following that drop is too high.

References

Bažant and Zhou (2002) "Why Did the World Trade Center Collapse?—Simple Analysis", *J. Eng. Mech.*, Vol. 128, No. 1, 2-6.

Bažant and Le (2008) “What Did and Did Not Cause Collapse of World Trade Center Twin Towers in New York?”, *J. Eng. Mech.*, Vol. 134, No. 10, 892-906

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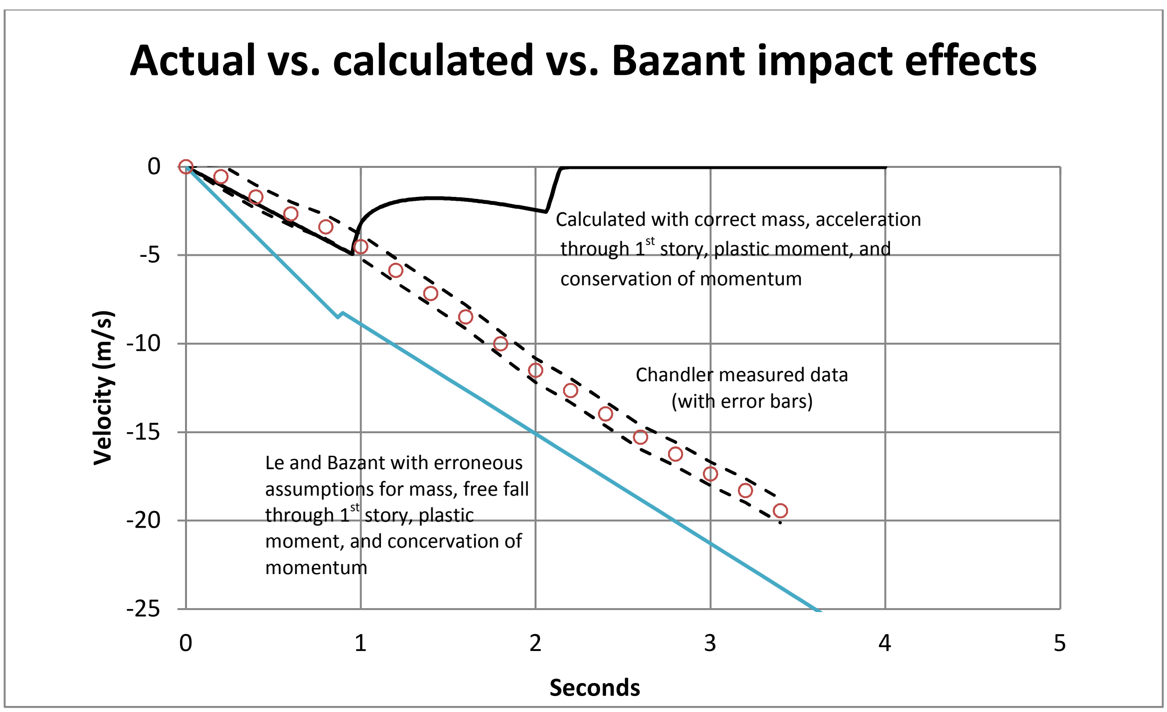
If the erroneous Le and Bazant assumption of the vanishing story and free fall through the 1st story is also replaced by the actual measured acceleration the below would be the result,



showing an arrest of the collapse in the second story of the fall. Le and Bazant embellish the kinetic energy by using nearly double the actual mass and acceleration while also diminishing the actual column energy absorption capacity by a factor of two.